Polar Equations and Transformations Calculus of a Single Variable X. Du

- Polar transformation from Cartesian plane:
  - $x = r\cos\theta \qquad y = r\sin\theta$
  - $r^2 = x^2 + y^2 \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$
- Slope: Use chain rule $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{d(r(\theta)\sin\theta)/d\theta}{d(r(\theta)\cos\theta)/d\theta} = \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta}$
- Area:
  - Use a Riemann sum of triangles: $A = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{2}\left(r(\theta_i^*)\right)^2\Delta\theta$
  - $A = \int_\alpha^\beta \frac{1}{2}r^2\,d\theta$
  - Also seen with a Jacobian transformation (discussed in multivariable calculus)
- Arc length: $s = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$
- Area of a surface of revolution
  - $S = 2\pi\int_\alpha^\beta r(\theta)\sin\theta\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$ about the x-axis
  - $S = 2\pi\int_\alpha^\beta r(\theta)\cos\theta\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$ about the y-axis